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Federated Optimization

Distributed Optimization Beyond the Datacenter

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Outline

- Optimization in Machine Learning
  - Empirical Risk Minimization
- Distributed Optimization
  - Motivation
  - Current challenges
- Federated Optimization
  - (possible) future of large-scale Machine Learning
  - Preliminary results
  - A lot of motivation for future research

Machine Learning Setting

- Space of input-output pairs $(x, y) \in \mathcal{X} \times \mathcal{Y}$
- Unknown distribution $P(x, y)$
- A relationship between inputs and outputs $P(y \mid x)$
- Loss function to measure discrepancy between predicted and real output $\ell(\hat{y}, y)$

- Define Expected Risk

$$E(\phi) = \int \ell(\phi(x), y) dP(x, y) = \mathbb{E}[\ell(\phi(x), y)]$$

Machine Learning Setting

- Ideal goal: Find ϕ^* such that,

$$\phi^*(x) = \arg\min_{\hat{y}} \mathbb{E}[\ell(\hat{y}, y) \mid x]$$

- But you cannot even evaluate $E(\phi)$

- Define Expected Risk

$$E(\phi) = \int \ell(\phi(x), y) dP(x, y) = \mathbb{E}[\ell(\phi(x), y)]$$

Machine Learning Setting

- We at least have IID samples $(x_i, y_i), i = 1, \ldots, n$

- Define Empirical Risk

$$E_n(\phi) = \frac{1}{n} \sum_{i=1}^{n} \ell(\phi(x_i), y_i) = \mathbb{E}_n[\ell(\phi(x), y)]$$

Machine Learning Setting

- First learning principle – fix a family $\mathcal{F}$ of candidate prediction functions

- Find Empirical Minimizer ϕ_n

$$\phi_n = \arg\min_{\phi \in \mathcal{F}} E_n(\phi)$$

- Define Empirical Risk

$$E_n(\phi) = \frac{1}{n} \sum_{i=1}^{n} \ell(\phi(x_i), y_i) = \mathbb{E}_n[\ell(\phi(x), y)]$$

Problem structure

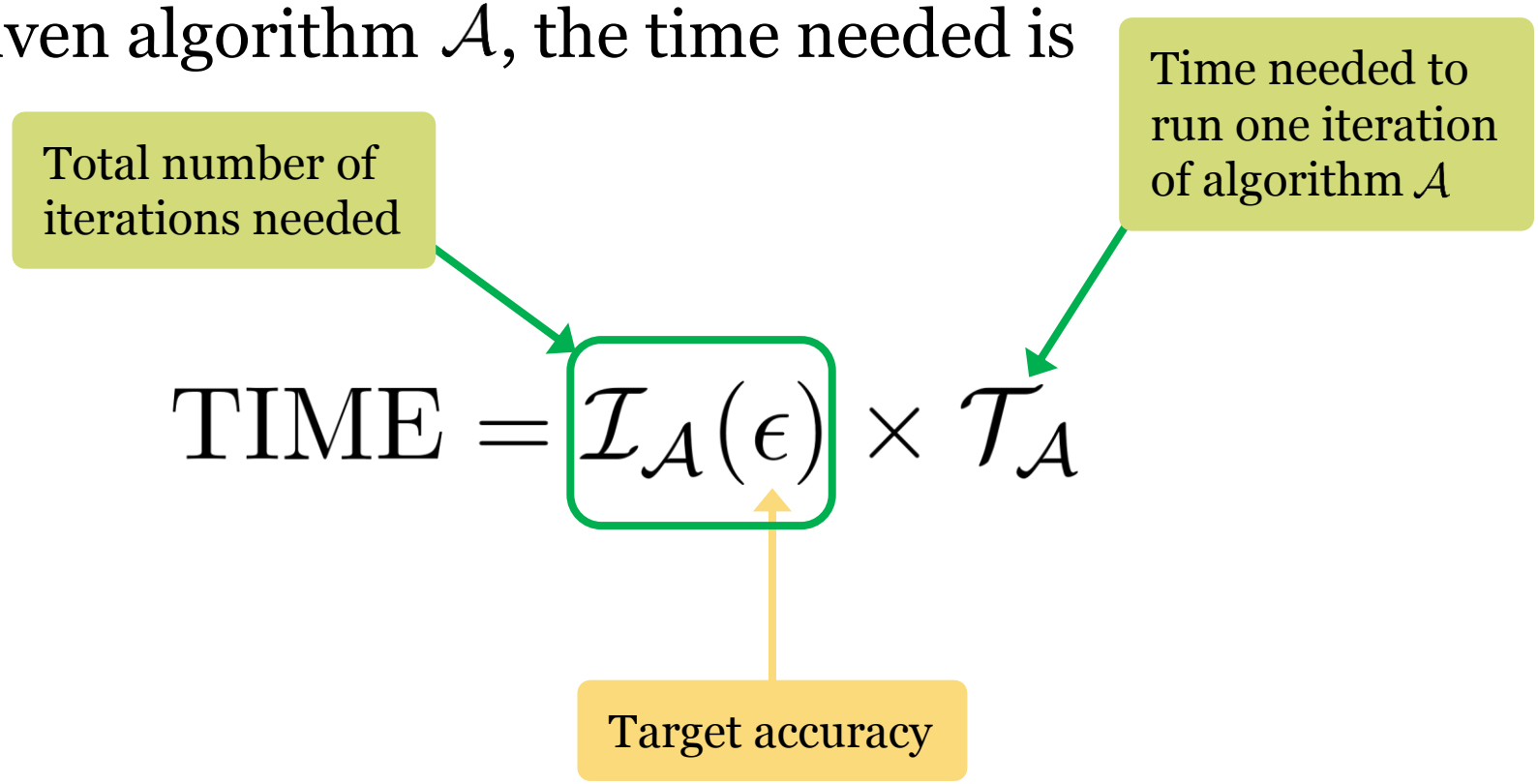
- Typical objective

$$\min_{x \in \mathbb{R}^d} \frac{1}{n} \sum_{i=1}^{n} f_i(x)$$

- Many recent fast algorithms for single computer
  - SGD (Stochastic Gradient Descent)
  - SDCA (Stochastic Dual Coordinate Ascent)
  - SAG (Stochastic Avergae Gradient)
  - SVRG/S2GD (Stochastic Variance Reduced Gradient)
  - SAGA (Improvement on SAG)
  - …

Traditional efficiency analysis

- Given algorithm $\mathcal{A}$, the time needed is



- Main trend – Stochastic methods
  - Big $\mathcal{I}_{\mathcal{A}}(\epsilon)$, small $\mathcal{T}_{\mathcal{A}}$

Distributed Optimization

Problem structure

- Original objective

$$\min_{x \in \mathbb{R}^d} \frac{1}{n} \sum_{i=1}^{n} f_i(x)$$

- Functions $f_i(x)$ described by n data points
- n can be millions
  - Articles in major newspaper
- n can be billions
  - Users of Facebook
- n can be trillions
  - Ad click-through rate predictions

Problem structure

- Original objective

$$\min_{x\in\mathbb{R}^d} \frac{1}{n}\sum_{i=1}^{n} f_i(x)$$

- Data distributed across K machines
  - Partition of $\{1,\dots,n\}$ to $\{\mathcal{P}_k\}_{k=1}^{K}$
  - Number of local data points $n_k = |\mathcal{P}_k|$
- Objective can be reformulated as

$$\min_{x\in\mathbb{R}^d} f(x) = \sum_{k=1}^{K} \frac{n_k}{n} F_k(x) = \sum_{k=1}^{K} \frac{n_k}{n} \cdot \frac{1}{n_k}\sum_{i\in\mathcal{P}_k} f_i(x)$$

Computer k has direct access only to function F_k

Algorithms for distributed optimization

- First trend
  - Naïve distributed variants of serial algorithms
  - Often inefficient; most time spent on communication

Communication time

- Let's say $v \in \mathbb{R}^{100}$

- Time to read v from memory (RAM)
  - 100 *ns*

- Time to send v to another machine
  - 500,000 *ns*

Distributed efficiency analysis

- There is lot of potential for improvement, if $c \gg \mathcal{T}_{\mathcal{A}}$ because most of the time is spent on communication

$$\text{TIME} = \mathcal{I}_{\mathcal{A}}(\epsilon) \times (c + \mathcal{T}_{\mathcal{A}})$$

Time for round of communication

Algorithms for distributed optimization

- First trend
  - Naïve distributed variants of serial algorithms
  - Often inefficient; most time spent on communication

- Second trend
  - Communication efficient algorithms
  - Idea: perform a lot of computation locally followed by a round of communication
  - Very useful in practice
  - DANE (Shamir et al., 2014)
  - CoCoA (Jaggi et al., 2014, 2015)
  - DiSCO (Zhang, Xiao, 2015) (Session WE16)

DANE

- A communication efficient algorithm for

$$\min_{x \in \mathbb{R}^d} f(x) = \sum_{k=1}^{K} \frac{n_k}{n} F_k(x)$$

Algorithm 1 Distributed Approximate Newton (DANE)

1: **Input:** starting point $\tilde{x}_0 \in \mathbb{R}^d$, regularizer μ
2: **for** $s = 0, 1, 2, \ldots$ **do**
3: Compute $\nabla f(\tilde{x}_s) = \frac{1}{n} \sum_{i=1}^{n} \nabla f_i(\tilde{x}_s)$ and distribute to all machines
4: For each node $k \in \{1, \ldots, K\}$, solve

$$x_k = \arg\min_{x \in \mathbb{R}^d} \left\{ F_k(x) - \left(\nabla F_k(\tilde{x}_s) - \nabla f(\tilde{x}_s) \right)^T x + \frac{\mu}{2} \|x - \tilde{x}_s\|^2 \right\}$$

5: Compute $\tilde{x}_{s+1} = \frac{1}{K} \sum_{k=1}^{K} x_k$
6: **end for**

DANE (necessary assumptions)

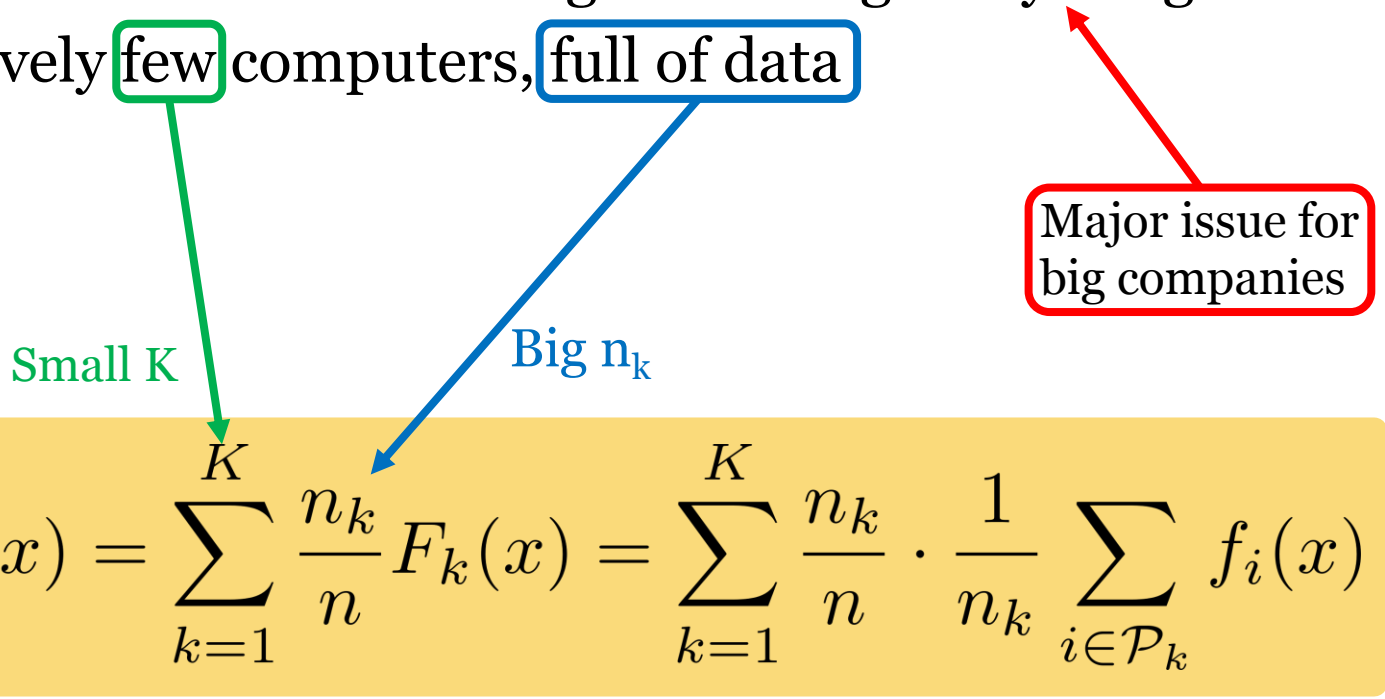
- Explicit
  - Each node has access to IID sample of data points
  - Each node has the same number of data points

- Implicit
  - Number of computers, K, (relatively) small
  - Size of data available locally, n_k, (relatively) big
  - In particular, $n_k \gg K$

- DANE relies on the assumptions heavily
  - Both in theory, and in practice

Distributed Optimization (today)

- Data are stored in datacenters
  - People worried about “Google knowing everything”
  - Relatively few computers, full of data



Federated Optimization

Federated Optimization (motivation)

- In order to even better protect privacy
  - Users keep their data on devices
  - Users provide computational power of devices
- Consequences
  - Large number of computers (devices)
  - Few data points on each device
  - Very different patterns in data on different devices

$$\min_{x \in \mathbb{R}^d} f(x) = \sum_{k=1}^{K} \frac{n_k}{n} F_k(x) = \sum_{k=1}^{K} \frac{n_k}{n} \cdot \frac{1}{n_k} \sum_{i \in \mathcal{P}_k} f_i(x)$$

DANE reminder

Algorithm 1 Distributed Approximate Newton (DANE)

1: **Input:** starting point $\tilde{x}_0 \in \mathbb{R}^d$, regularizer μ
2: **for** $s = 0, 1, 2, \ldots$ **do**
3: Compute $\nabla f(\tilde{x}_s) = \frac{1}{n} \sum_{i=1}^{n} \nabla f_i(\tilde{x}_s)$ and distribute to all machines
4: For each node $k \in \{1, \ldots, K\}$, solve

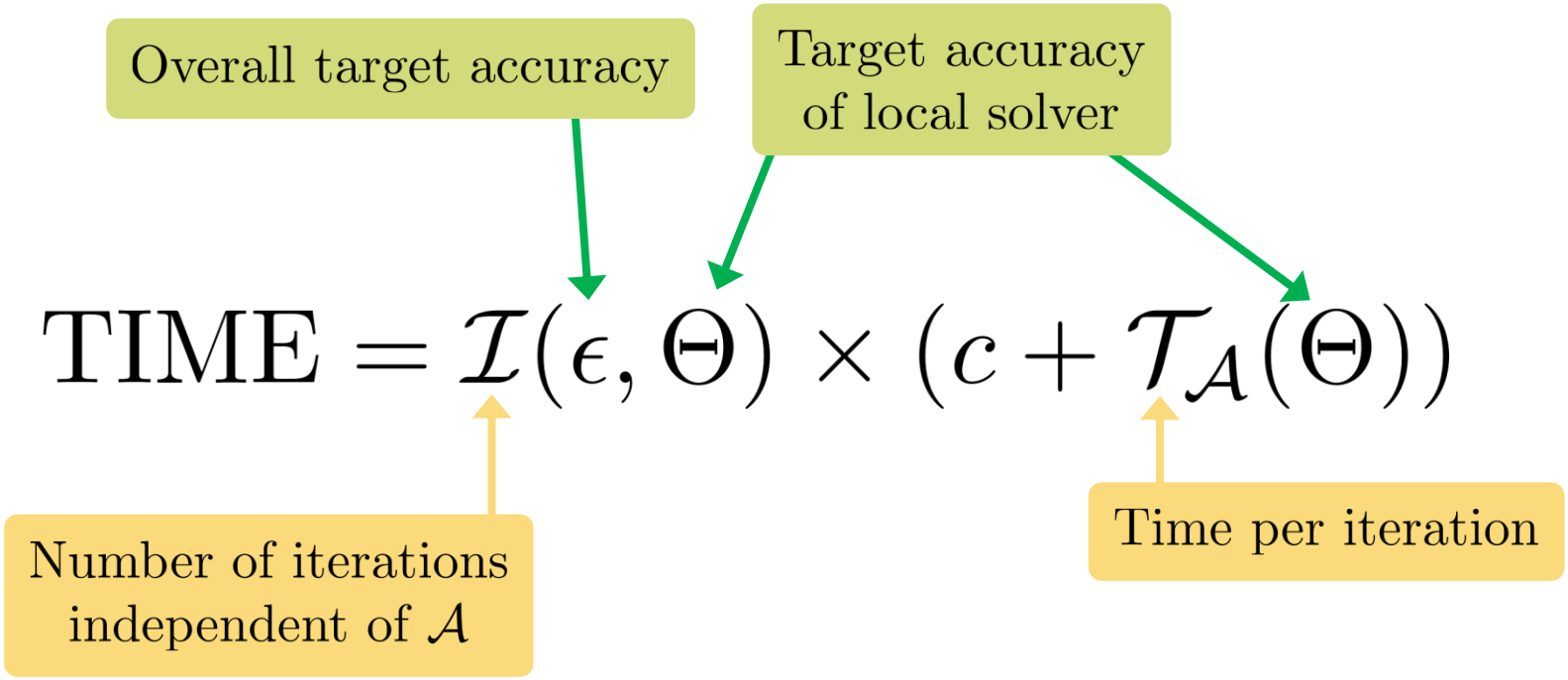
$$x_k = \arg\min_{x \in \mathbb{R}^d} \left\{ F_k(x) - \left(\nabla F_k(\tilde{x}_s) - \nabla f(\tilde{x}_s)\right)^T x + \frac{\mu}{2} \|x - \tilde{x}_s\|^2 \right\}$$

5: Compute $\tilde{x}_{s+1} = \frac{1}{K} \sum_{k=1}^{K} x_k$
6: **end for**

- Not stable / robust method
- But solving the subproblem approximately…

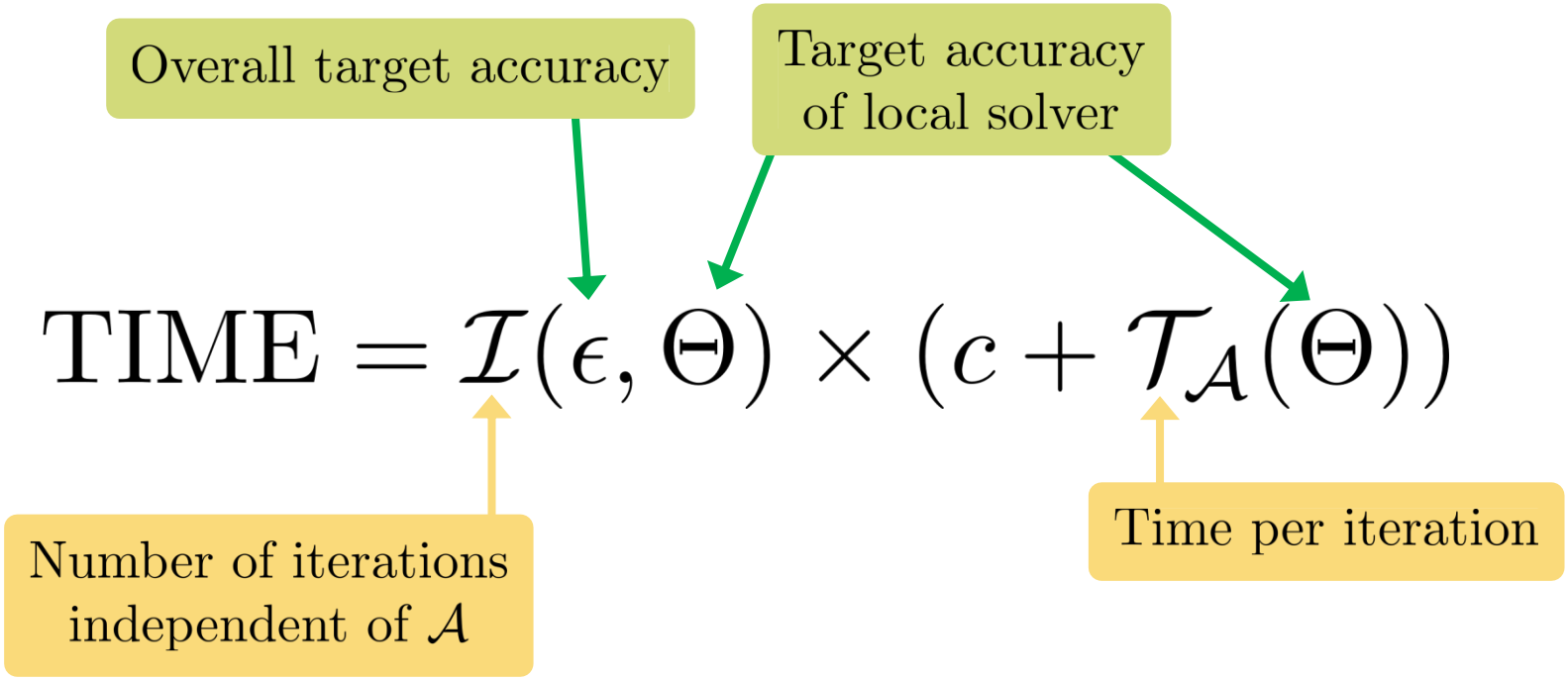
Efficiency analysis revisited

- If any algorithm $\mathcal{A}$ is used to solve the DANE subproblem to relative Θ accuracy



Efficiency analysis revisited – power

- If $\mathcal{A}$ chosen as Gradient Descent, run for 1 iteration
  - Equivalent to naïve distributed Gradient Descent
  - Greater flexibility with general subproblem



SVRG/S2GD algorithm

Algorithm 1 SVRG

parameters: m = max # of stochastic steps per epoch, h = stepsize
$\tilde{x}_0$ = starting point
for $j = 0, 1, 2, \ldots$ **do**
    Precompute $\nabla f(\tilde{x}_j) = \frac{1}{n} \sum_{i=1}^{n} \nabla f_i(\tilde{x}_j)$
    $x \leftarrow \tilde{x}_j$
    for $t = 0$ to $m - 1$ **do**
        Pick $i \in \{1, 2, \ldots, n\}$, uniformly at random
        $x \leftarrow x - h \left(\nabla f_i(x) - \nabla f_i(\tilde{x}_j) + \nabla f(\tilde{x}_j) \right)$
    end for
    $\tilde{x}_{j+1} \leftarrow x$
end for

Apply SVRG/S2GD as local solver in DANE

- Equivalent to

Algorithm 1 A distributed version of SVRG/S2GD

parameters: m = # of stochastic steps per epoch, h = stepsize, data partition $\{\mathcal{P}_k\}_{k=1}^K$, starting point $\tilde{x}_0$
for $s = 0, 1, 2, \ldots$ **do** ▷ Overall iterations
  Precompute $\nabla f(\tilde{x}_s) = \frac{1}{n}\sum_{i=1}^n \nabla f_i(\tilde{x}_s)$
  for $k = 1$ to K **do in parallel** over nodes k ▷ Distributed loop
    Initialize: $x_k = \tilde{x}_s$
    for $t = 1$ to m **do** ▷ Actual update loop
      Sample $i \in \mathcal{P}_k$ uniformly at random
      $x_k \leftarrow x_k - h\left(\nabla f_i(x_k) - \nabla f_i(\tilde{x}_s) + \nabla f(\tilde{x}_s)\right)$
    end for
  end for
  $\tilde{x}_{s+1} \leftarrow \tilde{x}_s + \frac{1}{K}\sum_{k=1}^K (x_k - \tilde{x}_s)$ ▷ Aggregate updates from nodes
end for

Improving the algorithm (sparsity)

- Sprasity
  - Data point in $\mathbb{R}^d$ contains only very small number of non-zero elements
  - Extremely common in large-scale applications
- Unbiasing stochastic gradients
  - Original update direction

$$\nabla f_i(x_k) - \nabla f_i(\tilde{x}_s) + \nabla f(\tilde{x}_s)$$

  - New update direction

$$S_k \left[\nabla f_i(x_k) - \nabla f_i(\tilde{x}_s)\right] + \nabla f(\tilde{x}_s)$$

Diagonal; ratio of global and local appearance frequencies

sparse vector $\in \mathbb{R}^{15}$

$$(1, 0, 0, 0, 3, 0, -2, 0, 0, 0, 0, 0, -1, 0, 0)$$

Improving the algorithm (sparsity)

- Sprasity
  - Data point in $\mathbb{R}^d$ contains only very small number of non-zero elements
  - Extremely common in large-scale applications
- Adaptive aggregation of updates
  - Original aggregation

$$\tilde{x}_{s+1} \leftarrow \tilde{x}_s + \frac{1}{K} \sum_{k=1}^{K} (x_k - \tilde{x}_s)$$

  - Updated aggregation

$$\tilde{x}_{s+1} \leftarrow \tilde{x}_s + A \sum_{k=1}^{K} \frac{n_k}{n} (x_k - \tilde{x}_s)$$

Diagonal; Inverse coordinate appearance on computers

Weighted average

sparse vector $\in \mathbb{R}^{15}$

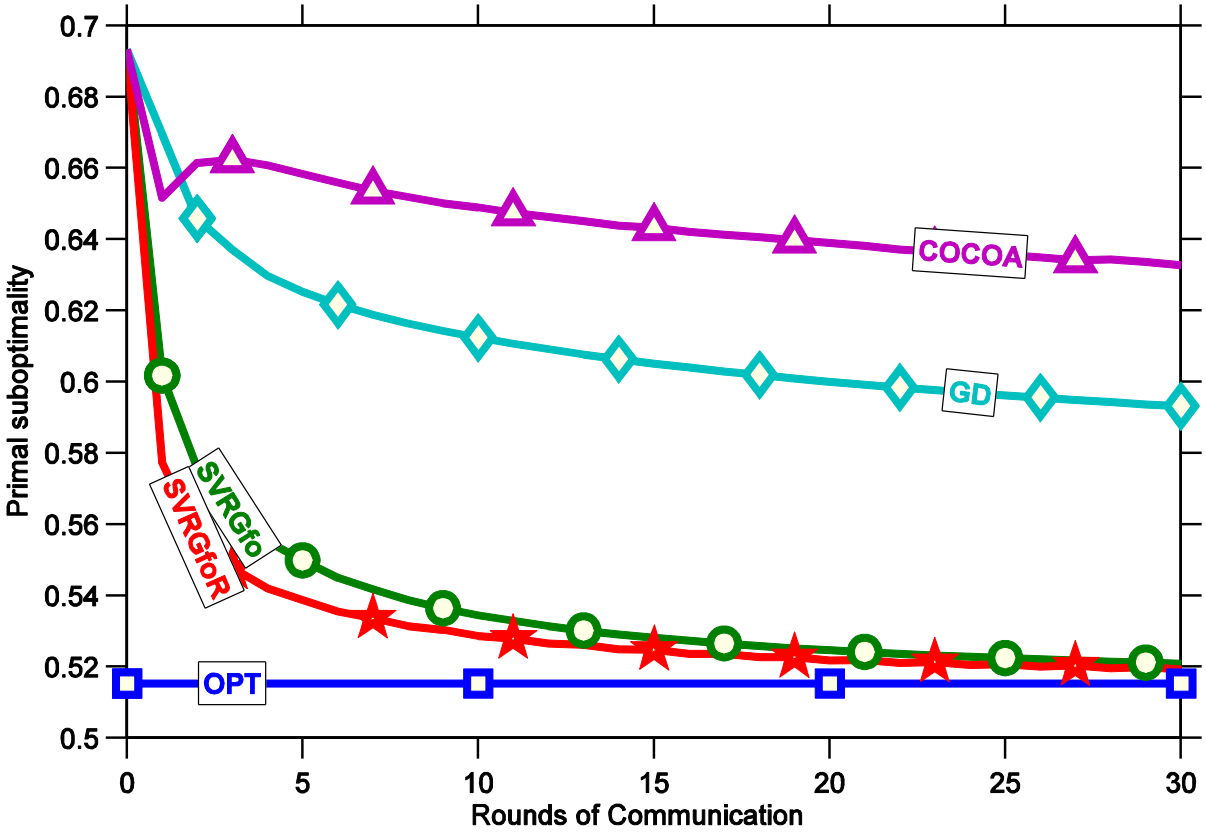
$$(1, 0, 0, 0, 3, 0, -2, 0, 0, 0, 0, 0, -1, 0, 0)$$

Experiments

- Setting
  - Data from public Google+ posts
  - Randomly chosen 10,000 users with 100 or more posts
  - Train/test split 75%/25%
  - Total 2,166,693 training posts
  - Extremes: 75 and 9,000 posts per user
  - See (simulate) each user as a separate computer
  - Simple bag-of-words language model
  - Dictionary of 20,000 most common English words
- Prediction task
  - Post has comment or not (binary classification)

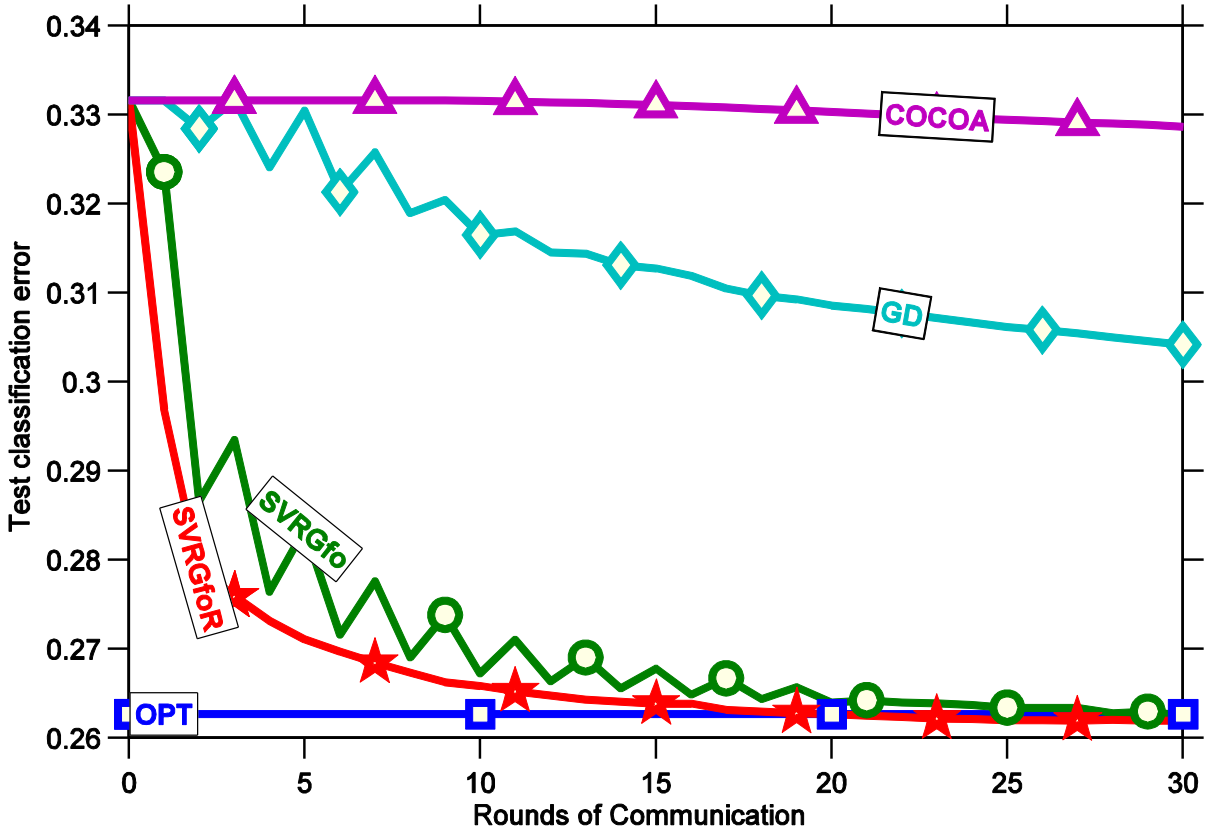
Experiments

- Optimization objective



Experiments

- Test error



Summary

- Federated Optimization
  - Very large number of computers K
  - Small number of data on each computer
  - Unbalanced – very different data sizes locally
  - Local data may be clustered; very far from IID

- Potentially industrial reality within few years
- Introduces many further questions

Future challenges

- More rigorous experiments
  - Larger problems
  - Deep learning
- Analysis missing!
- Need for good public dataset
  - With naturally per-user clustered data
- Asynchronous algorithms
- Differential privacy
  - Google cannot learn *anything*
- Many parts of ML pipelines need to be redesigned
  - Work for 1000+ people